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PREDICTION OF DENSITIES BASED ON SCARCE TRAFFIC FLOW INFORMATION

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Abstract

Traffic fluctuations are always evident in highways or urban arterial networks that consist of some signalized or unsignalized intersections. Traffic conditions may change as a result of changes in peak timing flows, miscellaneous incidents, variable weather etc. A constant challenge of traffic engineers and professional peoplethat are closely related to traffic control and management remains the identification of parts in which the traffic situation changes and the provision of information about traffic parameters.
Prediction of density parameter in short time intervals is important in lots of traffic modelling and control strategies of freeways and urban arterials. For more, the possession of short time density values for particular parts of the freeway segment, plays an importation role on providing drivers with information about events or traffic incidents. Not always the traffic flow amounts are possible to be measured in any part of the segment we are interested in. Thus maycome due to the lack of detector coverage or detecting defects even if it exists. The purpose of this paper is twofold. First is the development of a discrete model so called Cell Transmission Model (CTM) [1,2] that is analogue with approximation of the *LWR* hydrodynamic model of traffic flow. The second one is the integration of the Kalman Filter [3] to the mentioned model, in order to increase the accuracy of the modeled traffic densities. A Kalman filter (KF) is a recursive algorithm that uses only the previous time-step's prediction with the current measurement in order to make an estimate for the current state. KF does not require previous data to be stored or reprocessed with new measurements. At everyiteration, the KF minimizes the variance of the estimation error, making it an optimal estimator if linear and Gaussian conditions are satisfied. In order to highlight the difference between accuracies of the predictions of the densities

obtained by pure CTM model and by application of the Kalman Filter on it, a short highway segment with simple composition is chosen as object of study. The segment comprises of a ramp and the number o lanes are the same during its entire length.

Keywords–Cell Transmission Model, Time Step, Kalman Filter, density, error covariance.

INTRODUCTION

The Cell Transmission Model (CTM), which was originally obtained from the second order model of a traffic flow, was firstly developed by *Daganzo* [1, 2]. Since then it is very popular among researches due to its simplicity (it comprises a small number of traffic parameters) and its flexibility. As it was briefly described on the unit of discretized model, the *CTM* model discretizes the *LWR*[3, 4] model in every time unit choosing a time step Δt and road segment choosing length unit-cells of length l where these two parameters are chosen in order to fulfill the condition:

$$l = v_f.T_s \quad and \quad T_s < l/v_f$$

Where:

v_f is the free-flow speed or the average speed that vehicles develop during traveling under free flow conditions, and

T_s is the time step usually in seconds (T≈0.2-15 sec.)

Based on the *CTM* model, the number of vehicles in one cell is described according to a vehicle-conservation equation (1).

$$n_i(k+1) = n_i(k) + y_i(k) - y_{i+1}(k) \qquad (1)$$

Where:

$n_i(k+1)$is the number of vehicles in cell *i* at time step *k+1*

$n_i(k)$is the number of vehicles in cell *i* at time step *k,*

$y_i(k)$ is the number of vehicles entering from cell *i-1* to *i* during the time *k* and *k+1 and* is the flow that is determined by comparing the sending and receiving flow of cell i-*1* and *i*, respectively. According to *CTM*-first part of the model $y_i(k)$ is assumed to be the smallest of three values listed below:

$n_{i-1}(k)$, the number of vehicles in cell $_{i-1}$ at time *k*,

Q_i, the capacity flow into *i* for time interval *k*,

$N_i(k)$-$n_i(k)$ the amount of empty space in cell i at time k (this quantity ensures that the vehicular density on every section of the road remains below density).

As it seems from the above stated conditions, a cell can maximally receive a number of vehicles, which their adding should not exceed the maximal number of vehicles that can be present on it during time k, or a number of vehicles equal to the capacity flow or the a number of vehicles that the empty space of cell can accept during time k. Now equation for $y_i(k)$ takes the form as in (2):

$$y_i(k) = \min\,(n_{i-1}(k), Q_i(k), N_i(k) - n_i(k) \qquad (2)$$

1. MODEL OF A HIGHWAY SEGMENT

If we denote with $\rho_{i,}$(k) the density of a cell (uniform or non-uniform length), instead of the number of vehicles n_i on the a unit length cell, then we can bring equation *(1) to (3)* for density $\rho_i(k+1)$ of the cell i updated time step in *(k+1),* where T_s is the discrete time unit in seconds.

$$\rho_i(k+1) = \rho_i(k) + \frac{T_s}{L}(q_i(k) - q_{i+1}(k)) \qquad (3)$$

Fromabove equation we see that beside from the value of the density from the previous time step, density of a cell i also depends from the inter cell flows on the previous time step. Analyzing a highway partitioned in three cells (for the sake of simply illustration) with an on ramp and an off ramp, by assuming that the belonging cells can be in the free flow either in congested mode the densities on each cell can be written as in equations (4), (5) and (6).

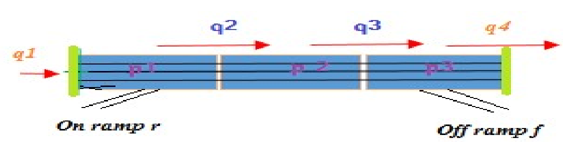

Fig. 1.CTM prsentation of Highway, inter cell flows

The densities on each cell are:

$$\rho_{i-1}(k+1) = \rho_{i-1}(k) + \frac{T_s}{L}(q_{i-1}(k) - q_i(k) + r(k)) \text{ or}$$
$$\rho_1(k+1) = \rho_1(k) + \frac{T_s}{L}(q_1(k) - q_2(k) + r(k)) \qquad (4)$$
$$\rho_i(k+1) = \rho_i(k) + \frac{T_s}{L}(q_i(k) - q_{i+1}(k)) \text{ or}$$
$$\rho_2(k+1) = \rho_2(k) + \frac{T_s}{L}(q_2(k) - q_3(k)) \qquad (5)$$

$$\rho_{i+1}(k+1) = \rho_{i+1}(k) + \frac{T_s}{L}(q_{i+1}(k) - q_4(k) - f(k)) \text{ or}$$
$$\rho_3(k+1) = \rho_3(k) + \frac{T_s}{L}(q_3(k) - q_4(k) - f(k)) \quad (6)$$

With the elaboration of the inter-cell flow law can be defined the expressions for the inter cell flows q_1, q_2 and $q_3$in the above equations.

Before the inter cell flows elaboration is given, a reasonable description of the congestion must be given further, since as we assumed above, the cells can be in either free or congested mode. Congestion is defined as the state of the traffic with high density rates, or with other words the density of that part of the highway expressed in cell is equal or higher than the critical density based on the fundamental diagram of relationship of flow and density. Referred to the mentioned diagram, can be noticed that the congested flow belongs to higher values of the density, above the critical density values where the flow drops down. That can be described with enormous number of vehicles travelling at low speeds and with short distance spaces between each other.

The common modes, used in analysis of researchers are the fully congested mode when the three cells are congested, denoted by *CCC* mode, and free flow mode when the three cells are in free flow mode, denoted by *FFF* mode. The other middle modes that are out of the scope of this paper are those with last one and two cells in congested mode, written by *FCC* and *FFC*, respectively. To emphasize the modes, the congested cells are highlighted further.

Now, for the *FFF* mode, the densities of the cells are lower than the critical density and the inter cell flows are as follows:

$$q_i(k) = \min\left(v_{fi-1} \cdot \rho_{i-1},\ Q_{i-1} w_i(\rho_J - \rho_i)\right) \text{ or} \quad (7)$$
$$q_2(k) = \min\left(\mathbf{v_{f1} \cdot \rho_1},\ Q_1 w_2(\rho_J - \rho_2)\right) = v_{f1} \cdot \rho_1$$
$$q_{i+1}(k) = \min\left(v_{fi} \cdot \rho_i, Q_i w_{i+1}(\rho_J - \rho_{i+1}),\right) \quad (8)$$
$$q_3(k) = \min\left(\mathbf{v_{f2} \cdot \rho_2},\ Q_2 w_3(\rho_J - \rho_3)\right) = v_{f2} \cdot \rho_2$$

In CCC mode, the densities of the cells are higher that the critical density, and the inter cell flows are:

$$q_i(k) = \min\left(v_{fi-1} \cdot \rho_{i-1},\ Q_{i-1} w_i(\rho_J - \rho_i)\right) \text{ or} (9)$$
$$q_2(k) = \min\left(v_{f1} \cdot \rho_1,\ Q_1 w_2(\rho_J - \rho_2)\right) = w_2(\rho_J - \rho_2)$$
$$q_{i+1}(k) = \min\left(v_{fi} \cdot \rho_i,\ Q_i w_{i+1}(\rho_J - \rho_{i+1}),\right) \quad (10)$$
$$q_3(k) = \min\left(v_{f2} \cdot \rho_2,\ Q_2 w_3(\rho_J - \rho_3)\right) = w_3(\rho_J - \rho_3)$$

After subtracting the expressions for inter cell flows in the equations of densities for the FFF mode, we have:

$$\rho_1(k+1) = \rho_1(k) + \frac{T_s}{L}(q_1(k) - v_{f1}.\rho_1(k) + r(k) \quad (11)$$

$$\rho_2(k+1) = \rho_2(k) + \frac{T_s}{L}(v_{f1}.\rho_1(k) - v_{f2}.\rho_2(k)) \quad (12)$$

$$\rho_3(k+1) = \rho_3(k) + \frac{T_s}{L}(v_{f2}.\rho_2(k) - q_4(k) - f(k) \quad (13)$$

And after subtracting the expressions for inter cell flows in the equations of densities for the *CCC*mode, we have:

$$\rho_1(k+1) = \rho_1(k) + \frac{T_s}{L}(q_1(k) - w_2\left(\rho_J - \rho_2(k)\right) + r(k)) \quad (14)$$

$$\rho_2(k+1) = \rho_2(k) + \frac{T_s}{L}(w_2(\rho_J - \rho_2(k)) - w_3(\rho_J - \rho_3(k)) \quad (15)$$

$$\rho_3(k+1) = \rho_3(k) + \frac{T_s}{L}(w_3(\rho_J - \rho_3(k)) - q_4(k) - f(k)) \quad (16)$$

2. KALMAN FILTER

Kalman filter-*KF* (*Kalman, 1960; Welch and Bishop 2001)* is a recursive data processing algorithm that uses only the previous time-step's prediction with the current measurement in order to make an estimate for the current state [3]. This means the *KF* does not require previous data to be stored or reprocessed with new measurements.

The Kalman Filter consists of a set of mathematical equations that provides an efficient recursive computation to estimate the state of a process by minimizing the mean of the squared error. The KF estimates the value of the variable x at any time (k+1), represented by a linear stochastic equation.

$$x_{k+1} = Ax_k + BX_k + w_k \quad (17)$$

Where: A (k) is matrix which relates the state a time interval *k* with the state at current time interval *k+1. B (k)* is matrix which relates the current state to the control inputX_k.

The random variable *w* represents noise in modelling process. It is assumed to be within normal probability distributions with zero mean and variance Q (Gaussian) as: $p(w)^{\sim}\ N(0,Q)$

The system measurement equation describes the relationship between system states and measurements. Acknowledging that measurements inevitably contain noise, the output equation is expressed as follows:

$$Z(k+1) = H \cdot x(k), \quad (18)$$

$\tilde{Z}(k)$is the measurement variable (outflow of vehicles from cell 3-measured by loop detector 2) *H (k)* is the output matrix, and *v (k)* is the measurement noise variable. The errors in estimating *a priori* and *a posteriori* states are defined as follows:

$$P_k^- = x_k - \hat{x}_k^- \quad (19)$$

$$\mathrm{P_k = x_k - \hat{x}_k} \quad (20)$$

The a priori and a posteriori estimate covariance is given by:

$$\mathrm{P_k^- = E\left[e_k^-, e_k^{-T}\right] = AP_{k-1} \cdot A^T + Q} \quad (21)$$

$$\mathrm{P_k = E[e_k, e_k{}^T) = (I - K_k H) \cdot P_k^-} \quad (22)$$

The KF estimates a posteriori state of the process using a linear combination of a priori state and a weighted difference between the actual measurement and the model measurement of the state.

$$\mathrm{\hat{x}_k = \hat{x}^-{}_k + K(z_k - H \cdot \hat{x}_k^-)} \quad (23)$$

$$\mathrm{K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}} \quad (24)$$

Based on the above equations, the *KF* process can be divided in two steps ore phases. The first step is the prediction step and the second step is the correction step.

2.1. APLICATION OF KALMAN FILTER FOR DENSITY PREDICTION

Predictive filtering techniques rely on a mathematical representation of the system. In this section, we discuss how to model a system's parameters and how to describe our indirect ability to inspect them. [6,7] A system is just like a black box, controlled by a set of parameters. To manipulate and understand a system, we need a mathematical model that describes these parameters and their behavior over time. In this section are described in detail the applied matrices to the algorithm of the *CTM-KF* model8,9]. It is necessary to recall the equations state space of *CTM* model first and then to do an interconnection of it with the *KF* algorithm equations. Since in our model, we are estimating the traffic densities of the three cells of the mentioned link, by the usage of the inputs values of the inflow q_1, outflow q_4 and the flow from on ramp, then the state space vector of our *KF* algorithm are the densities $x=[x_1,x_2,x_3]= [\rho_1, \rho_2, \rho_3]^T$,the input vector are $x_u=[q_1,0,q_4]^T$ *and*
$x_r=[qr,0,0]^T$

$$x_{k+1} = Ax_k + BX_{Uk} + w_k$$

$$\begin{pmatrix}\rho_1\\\rho_2\\\rho_3\end{pmatrix} = A \cdot \begin{pmatrix}\rho_1\\\rho_2\\\rho_3\end{pmatrix} + Bu \cdot \begin{pmatrix}q_1\\0\\-q_4\end{pmatrix} + Br \cdot \begin{pmatrix}r\\0\\0\end{pmatrix} \quad (25)$$

On this paper we are going to use the measurement of the outflow from the cell three, that corresponds to the flow q_4. Based on the fundamental diagram we model the traffic flow measurement through the densities on the last cell (ρ_3) and the free flow speed on cell 3 v_{f3} we will have $q_{out} = v_{f3} \cdot \rho_3$ that is consistent with the equation (26)

$$Z(k+1) = H \cdot x(k) \qquad (26)$$

$$H = [0\ 0\ v_{f3}]\ \ and\ x = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}, q_{out} - [0\ 0\ v_{f3}] \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix} \qquad (27)$$

Q -the model error covariance matrix which elements standard deviations of the density variables. The off diagonal elements are equal to zero while R is the measurement or output error covariance. In this seminar paper, the matrices Q and R are assumed to be constant.

$$Q = \begin{vmatrix} w & 0 & 0 \\ 0 & w & 0 \\ 0 & 0 & w \end{vmatrix}; R = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v \end{vmatrix} \qquad (28)$$

2.2. A NUMERICAL EXAMPLE

For the purpose of the demonstration of the CTM model, in this paper is performed a numerical example which is described below. The system of performance measurements of the traffic road networks of the Californian state *PeMs* is used for traffic collection data and is considered a freeway link for on the street “Broadway Avenue”, Stockton/San Francisco. The freeway consistsof three cells with different lengths with one on-ramp on the first cell. (*fig. 1 and 2)*

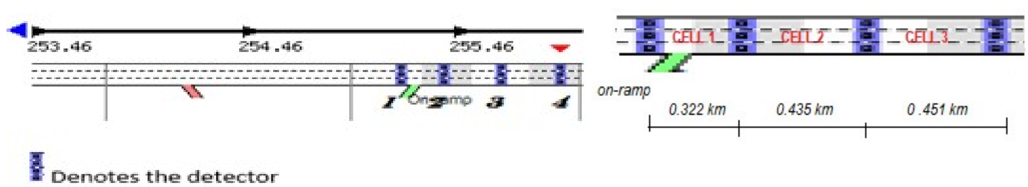

Fig. 2. Freewasegment with three cells

PeMs offers traffic measurements as flow, occupancy, speed, etc. that are collected by detectors every five minute intervals. The choice of the measurements for the applied link is done based on the demands that derive from the *CTM* model. Since we need to involve in the *CTM* model the free flow speed v_f[km/h], maximal flow or capacity Q_M*[veh/h],* critical density ρ_{cr}[veh/km] and jam density ρ_J*[veh/km]*, beside gathering the primary measurements, we are also pushed to do an calibration of the fundamental diagram to obtain the above mentioned measurements for every cell. Characteristic of calibration process of the *CTM* models calibration of *fundamental diagram* in order to reproduce as much as more accuracy results that lead to model results that resemble with real traffic conditions. Consequently, the process of calibration itself, in most cases and throughout this thesis is referred as calibration of the fundamental diagram.Another distinguished feature of the calibration to the cell transmission models, is that it provides the initial phase that precedes all other processes of modelling, while on other traffic or other scientific areas models the processof calibration is the last process used to be performed with the aim of improving the accuracy of the resultIn the frame of the calibration procedure of this seminar paper, the first step is to obtain the maximal amount of flow Q_M*[veh/h],* which provides the highest pint on the fundamental diagram with flow-density relationship. By applying the law of fundamental diagram for the relationship of the flow and density, we can obtain the value of the critical density by projecting the maximal point of the flow Q_M, to the horizontal axe 'x', which from the following fig. we can see that is equal to *59.9* (veh/km). Speed as important parameter of in the frame of calibration, presents the slope of the line drowned to the scattered plots, By the relationship for the flow and speed, $Q = \rho \cdot v$, we can calculate the free flow speed $v_f$corresponds to the critical density, as $vf = Q_M/\rho_{cr}$, which is equal to *86,2* (km/hr) (29).

$$v_f = \frac{Q_M}{\rho_{cr}} = \frac{5172,6}{59,9} = 86,2\ (\frac{km}{hr}) \qquad (29)$$

As an important part of the calibration is considered the estimation of the characteristic parameters that belong to the right side of the diagram or the congestion part, which are the jam density or the maximal density ρ_J*[veh/km]* and the backward speed *w[km/hr](18)*[8].

The jam density is determined by finding the outer point from the right side among the scattered plotfrom which can be seen the value of the critical density $\rho_J=248$ (veh/km).

The backward speed provides the rate at which the flow decreases while the density exceeds its critical value ρ_{cr}, which analogy as the free flow speed presents the slope of the line to the set of points of the right side diagram-congestion flow part. It has been calculated by the formula:

$$w = \frac{Q_M}{\rho_J - \rho_{cr}}\left[\frac{km}{hr}\right] = \frac{517,2}{248-59,9} = 27,5\left(\frac{km}{hr}\right) \qquad (30)$$

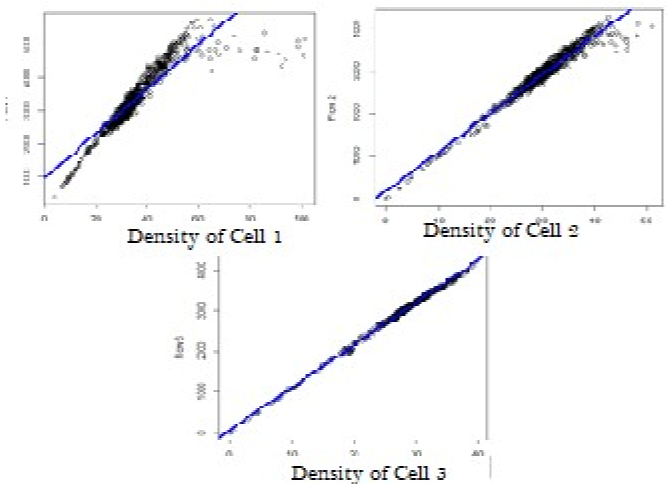

Fig. 4. Diagram for flow-density relationship of entire segment and for each cell

3. RESUSLTS AND CONCLUSIONS

Evaluation of the density values of cell is performed with discrete time intervals of $T_s=10$ seconds, where the initial values of the densities $\rho_0 = [\rho_{10}, \rho_{20}, \rho_{30}]^T$ and estimated covariance matrix P_o are assumed. T_s is chosen to be 10 second in order to fill the conditions $T<L/v_f$, for proper work with system matrices, otherwise there will be obtained negative values of density parameters. Prior to experiments realization, an initialization as a foregoing process is performed in order to estimate the evolution of traffic through artery from the "*zero point*". This "zero point" means the initial conditions of the whole segment, when all the cells are empty (density is zero), but the entry cell of the first segment feeds it with average traffic flow until is get a stable state of traffic.

For the purpose of the results evaluation, measured traffic densities for five minute intervals are used for comparison with the estimated densities with *CTM* model. The performance of the model was quantified by calculating the Mean Absolute Percentage Error (*MAPE)* given in (*31)*.

MAPE is a measure of prediction accuracy of a forecasting method in for regression problem. It usually expresses accuracy as a percentage, and is defined by the formula:

$$MAPE = [\frac{1}{n} \cdot \sum_{k=1}^{n} \left| 100 \frac{At-Ft}{At)}] \cdot 100 \right| \quad (31)$$

where A_t is the actual value and F_t is the predicted value. The difference between A_t and F_t is divided by the actual value A_t again. The absolute value in this calculation is summed for every forecasted point in time and divided by the number of fitted points *n*. Multiplying by 100% makes it a percentage error Related to the paper parameters MAPe i calculated as in *(32)*.

$$MAPE = [\frac{1}{n} \cdot \sum_{k=1}^{n} \left| 100 \frac{\rho_{mod}(k)-\rho_{meas}(k)}{\rho_{meas}(k)}] \cdot 100 \right| \quad (32)$$

Where, $\rho_{mod}(k)$ and $\rho_{meas}(k)$ are the estimated by model and measured values of density of cell during the k^{th} discrete time interval and n is the number of observations.

3.1. Excplication of Results

The *MAPE* results for *CTM* model for modes *FFF* -KF, for Cell 1, Cell 2 and Cell3, 2 %, 0.6 % and 1 % respectively, and for CCC-KF, 14% on the three cells. The results of the estimated values by *CTM* model against the measured values of traffic density are also graphically presented on the below figure *(fig. 4)*.

The best results of the performance are obtained with*CTM-FK(FFF)*.

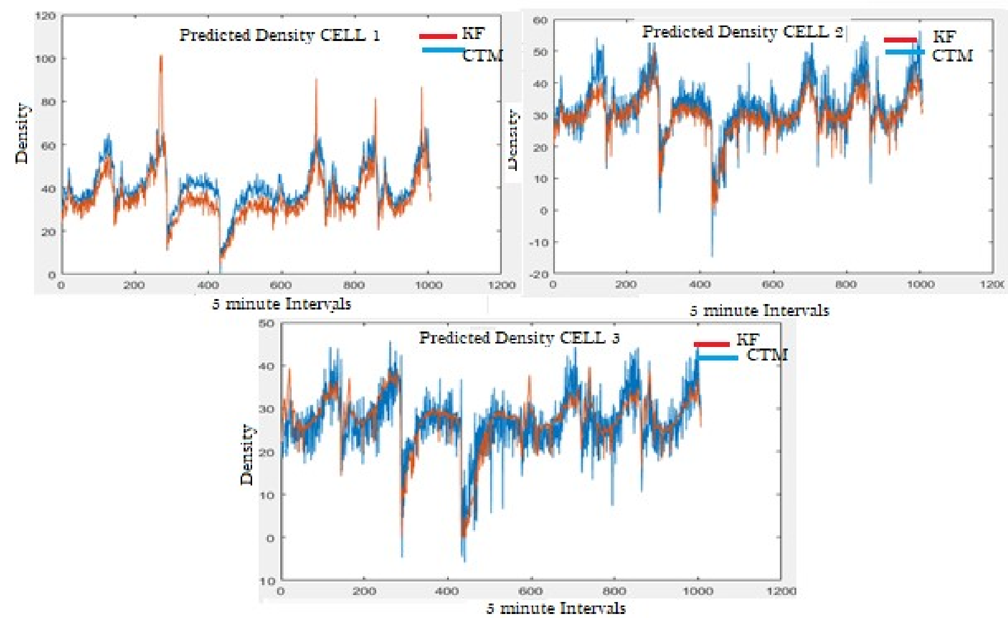

Fig. 4. Graphic results densities of KF FFF and measurements for each cell

3.2.Recomendations for further Reseaches

A further task toward the related paper future goal in terms of density forecasts remains their prediction even in the most complicated road networks such as those that have compositions of divergences and merges asurban arterieals with signaled intersections. It should be noted that before applying a recursive algorithm such as a kalman filter, in these cases the proper model of CTM should be formulated that includes not only the geometric configuration but also the traffic dynamics as well as the change of capacities due to the signals or even different number of lanes and so on.

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