DOI 10.20544/HORIZONS.B.03.1.16.P38
UDC 656.96:519.8(497.6)

COMBINED FUZZY AHP AND TOPSIS METHODFOR SOLVINGLOCATION PROBLEM[1]

Marko Vasiljević[1], Željko Stević
University of East Sarajevo
Faculty of Transport and Traffic Engineering Doboj, Vojvode Mišića 52
Doboj, Bosnia and Herzegovina
drmarkovasiljevic@gmail.com

Ilija Ćosić, Dejan Mirčetić
University of Novi Sad
Faculty of Technical Sciences, Trg Dositeja Obradovića 6
Novi Sad, Serbia

Abstract

Problems related to locations nowadays represent the wide field of interest, so methods contributing to their solving are already in day-to-day use. The aim of this paper is to create the model that mean the integration of fuzzy AHP and TOPSIS methods which enables us to estimate and valuate three potential locations for logistics centre construction in the territory of Republic of Srpska. Valuating is done on the base of six criteria compared between each other based on fuzzy triangular numbers and by applying Chang's extent analysis, what gives us high valued values for each criteria that greatly influence final rank of alternatives.

Keywords–Fuzzy AHP; Logistics centre; TOPSIS; location

[1] Original scientific paper

INTRODUCTION

The choice of the logistics centre location is based on an integrated decisions and risk methodology for the selection of the best locations. The integration process involves general steps as listed below:

The initial step forms a schedules collection and acceptable spatial alternatives. Step 1 defines a set of criteria for decision making, step 2 identifies the initial weight of the relevant criteria, step 3 uses the AHP as one of the techniques for multiple criteria decision making (MCDM), step 4 establishes a ranking list of alternatives using TOPSIS method. The final step selects the most suitable alternative.

Defining more alternative locations for logistics centres is necessary in order to perceive the different technical and technological sides of the issue and perform their comparison. Alternatives are first defined in the major hubs of traffic and goods flow (macro and micro level). So, a set of acceptable alternative locations is initially formed in order to choose optimum alternative in the following steps. For better comparison of allowed alternative it is necessary to define and quantify the spatial, traffic and geographic parameters and criteria.

So far, numerous MCDM approaches are applied or investigated to select the most suitable location of logistics centre. In [1] wasused fuzzy cluster analysis to LC location determination. Li et al.in [2] applied axiomatic fuzzy set and TOPSIS methodology to solve LC location problem. Fuzzy AHP and TOPSIS in [3] was using for evaluation Study on location selection of logistics centre.Erkayman [4] with help of fuzzy TOPSIS approach developed a model for selecting LC location. In[5] used fuzzy Additive ratio Assessment to determine LC best location.

AHP is often used in combination with other methods andfor the purpose of solving location problem, this paper uses the combination of methods of multi-criteria analysis. Fuzzy analytical-hierarchy process (FAHP) had been used for determination of significance of criteria, which compares criteria based on fuzzy scales for comparison, while Topsis method was used for alternatives' ranking.

CONVENTIONAL AHP METHOD

Analytic hierarchy process is created Thomas Saaty [6] and according to him [7] AHP is a measurement theory which is dealing with pairs comparing and which relies on expert opinion in order to perform the priority scale. With AHP according [8], it is possible to identify the relevant facts and connections existing between them. Parts of AHP method are problem decomposition, where the goal is located at the top, followed by criteria and sub-criteria, and at the end of the hierarchy are potential solutions, explained in more details by [9].

Some of the key and basic steps in the AHP methodology according to [10] are as follows: to define the problem, expand the problem taking into account all the actors, the objective and the outcome, identification of criteria that influence the outcome, to structure the problem previously explained hierarchy, to compare each element among them at the appropriate level, where the total of nx(n-1)/2 comparisons is necessary, to calculate the maximum value of own vector, the consistency index and the degree of consistency.

AHP in a certain way solves the problem of subjective influence of the decision-maker by measuring the level of consistency (CR) and notifies the decision maker thereof. If the level of consistency is in the range up to 0.10, the results are considered valid. Some authors take even greater degree of consistency as valid, which of course is not recommendable. This coefficient is recommended depending on the size of the matrix, so we may find in the papers of [11, 12] that the maximum allowed level of consistency for the matrices 3x3 is 0.05, 0.08 for matrices 4x4 and 0.1 for the larger matrices. If the calculated CR is not of the satisfactory value, it is necessary to repeat the comparison to have it within the target range.

CHANG'S EXTENT ANALYSIS

When it comes to decision making using fuzzy AHP method, various approaches were developed as expanded fuzzy AHP method based on triangular fuzzy numbers [13] fuzzy preference programming developed by [14] logarithmic fuzzy preference programming originated from the above mentioned access by its expanding, which was developed by [15].

The theory of fuzzy sets was first introduced by [16] whose application enables decision makers to effectively deal with the uncertainties. Fuzzy sets used generally triangular, trapezoidal and Gaussian fuzzy numbers, which convert uncertain fuzzy numbers numbers. Fuzzy set is a class of objects characterized by function of belonging, in which each object is getting a grade of belonging to the interval (0,1). Triangular fuzzy numbers, which were used in this work are marked as (lij, mij, uij). The parameters (lij, mij, uij) are the smallest possible value, the most promising value and highest possible value that describes a fuzzy event, respectively.

Let's assume that $X = \{x_1, x_2,...,x_n\}$ is number of objects, and $U = \{u_1, u_2,...,u_m\}$ is number of aims.

According to the methodology of extended analysis set up by Chang, for each object an extended goal analysis is made. Values of the extended analysis "m" for each object can be represented as follows:

$$M_{gi,}^{1} M_{gi,}^{2} M_{gi,}^{m} i = 1,2, \dots n., \quad (1)$$

where $M_{g,}^{j}$ $j = 1,2, \dots m.,$ are fuzzy triangular numbers. Chang's extended analysis includes following steps:

Step 1: Values of fuzzy extension for the i-ti object are given by the equation:

$$S_i = \sum_{j=1}^{n} M_{gi}^{j} \times \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} \quad (2)$$

In order to obtain expression

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} \quad (3)$$

it is necessary to perform additional fuzzy operations with "m" values of the extended analysis, which is represented by the following expressions:

$$\sum_{j=1}^{n} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \quad (4)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} M_{gi}^{j} = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \quad (5)$$

Then it is necessary to calculate the inverse vector:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} \left[\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right] \quad (6)$$

Step 2: Possibility degree Sb > Sa is defined:

$$V(S_b \geq S_a) = \begin{cases} 1, & if\ m_b \geq m_a \\ 0, & if\ l_a \geq u_b \\ \dfrac{l_a - u_b}{(m_b - u_b) - (m_a - l_a)}, & otherwise \end{cases} \quad (7)$$

where „d" ordinate of a largest cross-section in point D between µSa andµSb as shown in figure 1.

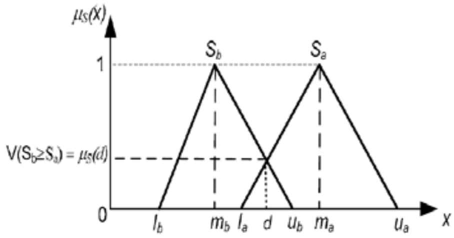

Fig. 1. Intersection between Sa and Sb

To compare S_1 and S_2, both values $V(S_1 \geq S_2)$ i $V(S_2 \geq S_1)$ are needed.

Step 3: Level of possibility for convex fuzzy number to be greater than „k“ convex number Si (i =1,2,...,k) can be defined as follows:

$$V(S_i \geq S_1, S_2, \dots, S_k) = \min V(S_i \geq S_k), = w'(S_i) \quad (8)$$

$$d'(A_i) = \min V(S_i \geq S_k), k \neq i, k = 1,2, \dots, n \quad (9)$$

The weight vector is given by the following expression:

$$W' = (\mathrm{d}'(A_1), \mathrm{d}'(A_2), \dots, \mathrm{d}'(A_n))^T, \quad (10)$$

Step 4: Through normalization, the weight vector is reduced to the phrase:

$$W = (\mathrm{d}(A_1), \mathrm{d}(A_2), \dots, \mathrm{d}(A_n))^T, \quad (11)$$

where W does not represent fuzzy number.

TOPSIS METHOD

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was first proposed by [17]. The basic idea for this method is to choose the alternative, which is as close to the positive ideal solution as possible and as far from the negative ideal solution as possible. The positive ideal solution is a solution with maximized benefit criteria and minimized cost criteria. The negative ideal solution is a solution, where the cost criteria are maximized and benefit criteria are minimized.

The following are the steps of the algorithm for solving the multi-criteria tasks of TOPSIS method:

Initial matrix $X = \|x_{ij}\|_{m \times n}$ (12)

Step 1 - normalization of the initial matrix:

$$\|X\| \to \|R\| \quad (13)$$

$$R = \|r_{ij}\|_{m \times n} \quad (14)$$

$$Rij=\frac{Xij}{\sqrt{\sum_{i-1}^{m} Xij^2}} \quad (15)$$

Step 2 - weighting of the normalized matrix:

$$\|R\| \rightarrow \|V\| \quad (16)$$

$$V = \|v_{ij}\| = \|W_j^{'} \cdot r_{ij}\| \quad (17)$$

Step 3 - forming the positive ideal and negative ideal solution:

A^+ - the positive ideal solution, which has all best features regarding all criteria:

$$A^+ = \left\{ \left(\max_i v_{ij} \mid j \in K' \right) i \left(\min_i v_{ij} \mid j \in K'' \right) \right\} = \left\{ v_1^+, v_2^+, ..., v_j^+, ..., v_n^+ \right\}, \quad i = \overline{1,m} \quad (18)$$

$K' \subseteq K \rightarrow K'$ is a subset of K consisting of *max* type criteria.

$K'' \subseteq K \rightarrow K''$ is a subset of K consisting of *min* type criteria.

A^- - the negative ideal solution, which has all worst features regarding all criteria:

$$A^- = \left\{ \left(\min_i v_{ij} \mid j \in K' \right) i \left(\max_i v_{ij} \mid j \in K'' \right) \right\} = \left\{ v_1^-, v_2^-, ..., v_j^-, ..., v_n^- \right\}, \quad (i = \overline{1,m}) \quad (19)$$

Step 4 - calculating the distance (Euclidean distance) of each alternative from the positive ideal and negative ideal solution:

S_1^+ - distance of an alternative from the positive ideal solution

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2} \quad (20)$$

S_1^- - distance of an alternative from the negative ideal solution

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2} \quad (21)$$

Step 5 - calculating the relative closeness of an alternative to the ideal solution:

$$C_i = \frac{S_i^-}{S_i^- + S_i^+} \quad (22)$$

$$0 \leq C_i \leq 1. \quad (23)$$

Step 6 - ranking of alternatives:

Ranking of Ci values arranged in descending order (from the highest to lowest value) corresponds to the ranking of Ai alternatives (from the best to worst).

NUMERICAL EXAMPLE

There is a number of criteria that can be studied in relation to the choice or ranking of alternatives. In order to define the relevant criteria, hierarchical structures were established, defining the group of high-level and lower-level criteria. The hierarchical structure of criteria used in this study for the choice of logistics center's location consists of 3 group criteria and 6 criteria (Table 1). Criteria were chosen in accordance with the standards for defining a set of criteria to be used in solving these problems.

Table 1. The hierarchical structure of the relevant criteria

Group criteria	**Criteria Level**	**Type**
Spatial	available surface	Numerical
	Land price	
Geographic	geographical location	Linguistic
	macro-micro level of location	
Traffic	affiliation to the form of transportation	Numerical
	approach ways accessibility of transport equipment to the logistics center	Linguistic

Following the methodology described for decision making to get the required results is necessary to perform criteria comparison on the basis of fuzzy triangular numbers, as shown in Table 3. The comparison was made based on the scale shown in Table 2 as defined in [13].

Table 2.Triangular fuzzy scale table

Linguistic Scale	**Triangular Fuzzy Scale**	**Triangular Fuzzy Reciprocal Scale**
Just equal	(1, 1, 1)	(1, 1, 1)
Equaly important	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly more important	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly more important	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strongly more	(2, 5/2, 3)	(1/3, 2/5,

important		1/2)
Absolutely more important	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

By comparing them, weight value criteria is determined, and that criteria plays very important role in the further implementation of methods, because on the base of these values the optimal solution is determined.

Table 3.Comparison of criteria on the base of triangular numbers

	C₁	**C₂**	**C₃**	**C₄**	**C₅**	**C₆**
C 1	(1,1,1)	(1,3/2,2)	(1/2,1,3/2)	(1/2,1,3/2)	(1,3/2,2)	(3/2,2,5/2)
C 2	(1/2,2/3,1)	(1,1,1)	(2/3,1,2)	(2/3,1,2)	(1,1,1)	(1/2,1,3/2)
C 3	(2/3,1,2)	(1/2,1,3/2)	(1,1,1)	(1,1,1)	(1/2,1,3/2)	(1,3/2,2)
C 4	(2/3,1,2)	(1/2,1,3/2)	(1,1,1)	(1,1,1)	(1/2,1,3/2)	(1,3/2,2)
C 5	(1/2,2/3,1)	(1,1,1)	(2/3,1,2)	(2/3,1,2)	(1,1,1)	(1/2,1,3/2)
C 6	(2/5,1/2,2/3)	(2/3,1,2)	(1/2,2/3,1)	(1/2,2/3,1)	(2/3,1,2)	(1,1,1)

To determine Fuzzy combination expansion for each one of the criteria, first we calculate $\sum_{j=1}^{n} M_{gi}^{j}$ value for each row of the matrix.

C_1 = (1+1+1/2+1/2+1+3/2; 1+3/2+1+1+3/2+2; 1+2+3/2+3/2+2+5/2) = (5.5; 8; 10.5) etc.

The $\sum_{i=1}^{n} \sum_{j=1}^{n} M_{gi}^{j}$value is calculated as:

(5.5; 8; 10.5) + (4.333; 5.667; 8.5) + (4.667; 6.5; 9) + (4.667; 6.5; 9) + (4.333; 5.667; 8.5) + (3.533; 4.833; 7.667) = (27.033; 37.167; 53.167)

Then, $S_i = \sum_{j=1}^{n} M_{gi}^{j} \times \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$:

S_1 = (5.5; 8; 10.5) x (1/53.167; 1/37.167; 1/27.033) = (0.103; 0.215; 0.388) etc.

Now, the V values (preference order) are calculated using these vectors.

$V(S_1 \geq S_2) = 1 V(S_1 \geq S_3) = 1 V(S_1 \geq S_4) = 1 V(S_1 \geq S_5) = 1$

$V(S_1 \geq S_6) = 1$

$$V(S_2 \geq S_1) = \frac{0.103 - 0.314}{(0.152 - 0.314) - (0.215 - 0.103)} = 0.770$$

$$V(S_2 \geq S_3) = \frac{0.088 - 0.314}{(0.152 - 0.314) - (0.175 - 0.088)} = 0.907$$

$$V(S_2 \geq S_4) = \frac{0.088 - 0.314}{(0.152 - 0.314) - (0.175 - 0.088)} = 0.907$$

$V(S_2 \geq S_5) = 1$ $V(S_2 \geq S_6) = 1$ etc.

The priorities of weights are calculated using:

d' = (C_1) min (1; 1; 1; 1; 1) = 1
d' = (C_2) min (0.770; 0.907; 0.907; 1; 1) = 0.770
d' = (C_3) min (0.852; 1; 1; 1; 1) = 0.852
d' = (C_4) min (0.852; 1; 1; 1; 1) = 0.852
d' = (C_5) min (0.770; 0.907; 0.907; 1; 1) = 0.770
d' = (C_6) min (0.680; 0.902; 0.813; 0.813; 0.902) = 0.680

After the equation is applied (10), weight values are obtained, and from the equation (11) normalized weights of criteria are received:
W'=(1; 0.770; 0.852; 0.852; 0.770; 0.680) W=(0.20; 0.16; 0.17; 0.17; 0.16; 0.14)

After determination of criteria, it is clear that the first criteria ie. available surface for this study represents the most important criteria during the evaluation of potential locations. By applying previously described steps of Topsis method, results represented by the following figure were obtained.

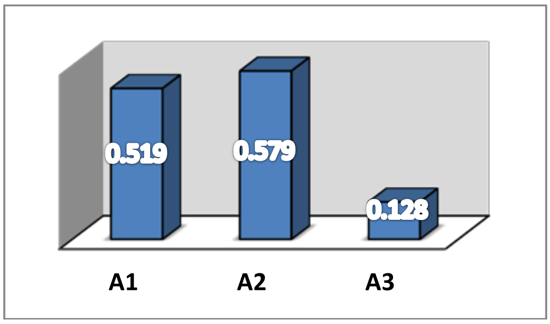

Fig. 2. Ranking of alternatives

On figure 2, it is visible that the alternative no. 2 (location Šamac) has the highest value and represent the most suitable location according to previously conducted steps.

CONCLUSION

The relative weight of the criteria has a major influence on the final decision to solve problems using methods of multi-criteria analysis. This influence can be best reflect the the comparison with [18], where the same problem is solved by classical AHP method and where the highest priority has the second criterion, ie. the price of land, and rank different alternatives ie. first location is the most suitable.In this case, with the increase of importance first criteria ie. available surface first alternative becomes the best solution as compared to the other has the biggest space about 40 hectares.

REFERENCES

[1] Ren, Yong-chang, et al. "Fuzzy Cluster Analysis of Regional City Multi-level Logistics Distribution Center Location Plan." *Quantitative Logic and Soft Computing 2010*. Springer Berlin Heidelberg, 2010. 499-508.

[2] Li, Ye et. al. "Selection of logistics center location using Axiomatic Fuzzy Set and TOPSIS methodology in logistics management." *Expert Systems with Applications* 38.6 (2011): 7901-7908.

[3] Wang, Shengyuan, and Peide Liu. "The evaluation study on location selection of logistics center based on fuzzy AHP and TOPSIS." *Wireless Communications, Networking and Mobile Computing, 2007. WiCom 2007. International Conference on*. IEEE, 2007.

[4] Erkayman, Burak, et al. "A fuzzy TOPSIS approach for logistics center location selection." *Journal of Business Case Studies (JBCS)* 7.3 (2011): 49-54.

[5] Turskis, Zenonas, and Edmundas Kazimieras Zavadskas. "A new fuzzy additive ratio assessment method (ARAS-F). Case study: The analysis of fuzzy multiple criteria in order to select the logistic centers location." *Transport* 25.4 (2010): 423-432.

[6] Saaty, T. L. “The Analytic Hierarchy Process”, Mc Graw-Hill, NewYork 1980.

[7] Saaty, Thomas L. "Decision making with the analytic hierarchy process."*International journal of services sciences* 1.1 (2008): 83-98.

[8] Saaty, T. L. What is the analytic hierarchy process? 1988. (pp. 109-121). Springer Berlin Heidelberg.
[9] Saaty, Thomas L. "How to make a decision: the analytic hierarchy process."*European journal of operational research* 48.1 (1990): 9-26.
[10] Vaidya, Omkarprasad S., and Sushil Kumar. "Analytic hierarchy process: An overview of applications." *European Journal of operational research* 169.1 (2006): 1-29.
[11] Lee, A. H. et. al. A fuzzy AHP and BSC approach for evaluating performance of IT department in the manufacturing industry in Taiwan. *Expert systems with applications*, (2008). *34*(1), 96-107.
[12] Anagnostopoulos, K. P. et. al. "Using the fuzzy analytic hierarchy process for selecting wastewater facilities at prefecture level."*European Water* 19.20 (2007): 15-24.
[13] Chang, D. Y. Applications of the extent analysis method on fuzzy AHP. European journal of operational research, 95(3), (1996), pp. 649-655.
[14] Mikhailov, L. Deriving priorities from fuzzy pairwise comparison judgements. Fuzzy sets and systems, 134(3), (2003), pp. 365-385.
[15] Wang, Y. M. and Chin, K. S. Fuzzy analytic hierarchy process: A logarithmic fuzzy preference programming methodology. International Journal of Approximate Reasoning, 52(4), (2011), pp. 541-553.
[16] Zadeh, L.A., Fuzzy sets. Information and Control, 8(3), (1965), pp. 338-353.
[17] Hwang, C. L. and Yoon, K. Multiple attributes decision making methods and applications. (1981) Berlin: Springer.
[18] Stević, Željko, et al. "The selection of the logistics center location using AHP method." (2015): 21-23.