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# A MODEL OF COMPETITION AMONG TELECOMMUNICATION SERVICE PROVIDERS BASED ON REPEATED GAME[1]

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**Abstract**

In a highly competitive telecommunication market each service provider (SP) tends to improve his position in relation to competitors. In order to achieve higher market share and consequently higher profit, a SP in addition to improving quality of service should implement efficient pricing and marketing strategies. In this paper we propose a repeated game approach for modelling the competition among rational SPs with similar reputations in the same telecommunication market. We define the service demand and payoff functions as SPs' profit functions. This model may be applicable for obtaining the optimal trade-off between the available strategies, considering current reputation in the market and concurrent SP's actions.

***Keywords*–*Nash equilibrium;payoff;price; profit; repeated game; service provider***

## INTRODUCTION

Pricing telecommunication services is determined by various factors, among which the most important are service demand and competitiveness of service providers (SPs) in a telecommunication market. Therefore demand function has to be defined precisely and should take into account the service price offered by the observed provider as well as prices of the same service offered by his competitors [1]. Relations between SPs are complex because they don't have always strictly opposing interests but also have to cooperate.

Game theory is a promising solution for modelling competition in growing markets, such as telecommunication markets. Repeated games are suitable for modelling situations where players repeatedly interact with each

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other. In such situations, a player can condition his behaviour at each point in the game on the other players' past behaviour [2], [3]. In this paper we observe two SPs as players in telecommunication market. We use repeated game model for analysing competition between SPs and Nash equilibrium concept for determining the optimal strategies.

The paper is organized in the following way. In Section 2 we defineservice demand and service providers' profit functions considering the service offered by two SPs with similar reputations in the same telecommunication market. In Section 3 basic assumptions in game theory and components of games are briefly presented. In the same Sectionrepeated game model for analysing competition between two service providers is described and we also discuss the Nash equilibrium of the repeated game example. Conclusion is given in the Section 4.

## SERVICE DEMAND AND SERVICE PROVIDERS' PROFIT FUNCTIONS

For the same or similar services there ismultiple interdependence between a price and demand for a service. Higher demand for the service allows the service provider to increase the service price and to simultaneously achieve higher revenue from this service. On the other hand, if demand for the service decreases, the price must be reduced in order to attract new customers. Competition between service providers also affects pricing. Customers will prefer a provider offering the service with the best quality/price relation. If a provider reduce the cost of a service he will attract more customers and will earn more revenue, which will affect the income of other providers in the market that offer the same or a similar service. The expected reaction of competitors is to reduce the price of their services to retain customers [1].

For the purpose of this study, wedefine a total demand for a service in the market with two service providers - SP*i* and SP*j*:

$$D_t = D_i + D_j \tag{1}$$

$$D_i = D_0^i - b_i^i p_i + b_i^j p_j + D_m \tag{2}$$

$$D_j = D_0^j - b_j^j p_j + b_j^i p_i + D_m \tag{3}$$

$$D_t = D_0^i + D_0^j + p_i\left(b_j^i - b_i^i\right) + p_j\left(b_i^j - b_j^j\right) + 2D_m \tag{4}$$

where:

$D_0^i$ - positive constant representing the total demand of $SP_i$'sservice if the service was free

$p_i$ - service price set by $SP_i$

$b_i^i$ - sensitivity of $SP_i$'s customersto its price change

$b_i^j$ - sensitivity of $SP_i$'s customersto price of $SP_j$. It indicates what effect of changing the price $p_i$compared to $p_j$will have on the customer intention to purchase the given service.

$D_m$ - increase in demand caused by marketing promotions.

$SP_i$'sprofit:

$$\Pi_i = p_i \cdot D_i - c_i \tag{5}$$

$c_i$ - operational cost

$$c_i = c_b + c_m \tag{6}$$

$c_b$ - basic cost

$c_m$ - marketing cost

We assume the same marketing cost for both SPs:

$$c_m = \frac{1}{2} c_s a_s^2 \tag{7}$$

$c_s$ - SP's cost parameter

$a_s$ - SP's advertising effort

Profit change is defined as follows:

$$\Delta\Pi_i = \begin{cases} \dfrac{\Pi_i^1 - \Pi_i^0}{\Pi_i^0} & \text{if } p_i \neq p_j \vee c_i \neq c_j \\ 0 & \text{if } p_i = p_j \wedge c_i = c_j \end{cases} \tag{8}$$

$\Pi_i^0$ – initial profit

## REPEATED GAME SETTING FOR MODELLING COMPETITION BETWEEN TWO SERVICE PROVIDERS

Game theory is a field of applied mathematics that describes and analyses interactive decision making situations. It consists of a set of analytical tools that predict the outcome of complex interactions among rational players [3].

Basic components of a game are players, the possible strategies of the players and consequences of the strategies, i.e. outcomes or payoffs. The players try to ensure the best possible consequence according to their preferences. The preferences of a player can be expressed either with a utility function, which maps every consequence to a real number, or with preference relations, which define the ranking of the consequences.

The most fundamental assumption in game theory is rationality. Rational players are assumed to maximize their payoff. If the game is not deterministic, the players maximize their expected payoff [3], [4]. It is also assumed that all players know the rules of the game well.

A game describes what strategies the players can take and what the consequences of these strategies are. The solution of a game is a description of outcomes that may emerge in the game if the players act rationally and intelligently. Generally, a solution is an outcome/payoff from which no player wants to deviate unilaterally.

When engaged in a repeated situation, players must consider not only their short-term outcomes but also their long-term payoffs. The general idea of repeated games is that players may be able to deter another player from exploiting his short-term advantage by threatening punishment that reduces his long-term payoff [5], [6].

For the purpose of this study we observe two service providers as players in the repeated game. We observe the price of an individual service offeredby SP either individuallyor bundledwith other services. For SPs with similar reputation in the telecommunication market, we suppose three possible strategies:

- A – lower price,
- B – lower price combined with marketing efforts and
- C – additional content for the same price combined with marketing efforts.

The assumption is that the strategy C, in terms of individual service, means even a lower price of the service when it is sold bundled with other facilities and therefore the price is set lower for strategy C compared to strategies A and B. We suppose that choosing strategy C means price reduction of 10% compared to the start, and choosing A or B means 5% lower price in comparison to the starting price.

In this study, we set the initial price to 3MU (money unit) for both SPs and the initial demand $D_1 = D_2 = 0.4$. We have also assumed the following numerical values: $b_1^1 = b_2^2 = 0.4$, $b_1^2 = b_2^1 = 0.2$ and $c_1 = c_2 = 0.2$MU. For these values, we have calculated the difference between the realized and the initial profit expressed in percentages for all possible strategies combination for both SPs. (Table 1). In further text the difference between the realized and the initial profit is denoted as payoff.

Table 1. Service providers strategies and the corresponding payoffs

| | | **2** | | |
|---|---|---|---|---|
| | | A | B | C |
| | A | (0, 0) | (3, 8) | (-6, 17) |
| **1** | B | (8, 3) | (0, 0) | (0, 17) |
| | C | (17, -6) | (17, 0) | (0, 0) |

With the aim of modelling the competition among rational service providers with similar reputations in the same telecommunication market we assume the following repeated game setting:

- In the first stage of the game, the two service providers simultaneously choose among their actions, observe the outcome, and then in the second stage playthe static game again.
- The payoffs are simply the discounted average from the payoffs in each stage. That is, let $p_{i1}$represent SP*i*'s payoff at the first stageand $p_{i2}$represent his payoff at the second stage. Then SP*i*'s payoff from the multi-stage game is $u_i = p_{i1} + \delta\, p_{i2}$, where $\delta \in (0, 1)$is the discount factor.

According to the proposed game setting and Table 1, it is reasonable to suppose the following course of the game:

- $SP_1$: play Aat the first stage. If the outcome is (A, A), play Bat the second stage, otherwise play A with probability of 3/11 and B with probability of8/11at the second stage.
- $SP_2$: play Cat the first stage. If the outcome is (A, C) or (C, C), play Aat the second stage, otherwise play A with probability of 3/11 and B with probability of 8/11 at the second stage.

Since the strategies at the second stage specify playing Nash equilibrium profiles for all possible second stages, optimal strategy profile for both SPs include only strategies A and B. Both providers receive higher payoffs if they choose different strategies, i.e. (A, B) or (B, A), which is given in Table 2.This results in playing mixed strategy Nash equilibrium at the second stage.

Table 2. Service providers strategies and the corresponding payoffs at the second stage of the repeated game

| | | **2** | |
|---|---|---|---|
| | | A | B |
| | A | (0, 0) | (3, 8) |
| **1** | B | (8, 3) | (0, 0) |

In this case $SP_1$ will not deviate from the strategy profile (A, B) if $8 + \delta*24/11 \leq 3 + \delta*8$, i.e. $\delta \geq 55/64$. Similarly, $SP_2$ will not deviate if $8 + \delta*24/11 \leq 3 + \delta*3$, i.e. $\delta \geq 55/9$.

We conclude that the strategy profile specified above is a subgame perfect Nash equilibrium if $\delta \geq 55/9$. In effect, SPs can attain the non-Nash efficient outcome at the first stage by threatening to revert to the worst possible Nash equilibrium at the second stage.

## CONCLUSION

Game theory is especially convenient means for analysing competition between service providers offering the same or similar service in the same telecommunication market.In this paper we have researched possibilities of using game theoretic approach for computing competitive prices and profits of two service providers. We have observed two service providers, with similar reputation in the market, as players of the repeated game. For the defined repeated game setting we have discussed and determined a subgame perfect Nash equilibrium.

In further research we will consider possibilities of modelling the competition among service providers with different reputations in the same telecommunication market as well as the impact of different customers' behaviour.

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